THE OPTIONCITY CALCULATOR
VERSION 1.1
TUTORIAL AND HELP

Copyright © 2001-2006 by OptionCity.net. All rights reserved.

Last revision to this document: May.9, 2006

Algorithms, Conceptual Design, and this Tutorial by Alan L. Lewis
User Interface Design and Programming by Milind Sardesai

Note. This document is **OptHelp.pdf** . It is found in the same directory as your Calculator executable. You may view it directly from your Adobe Acrobat reader (Acrobat Version 5.0 and higher recommended, available at http://www.adobe.com), instead of from the Help tab of the Calculator.

CONTENTS

I. Tutorial 2
I.1 Overview 2
I.2 The Option Models and their Parameters 6
I.3 Working with the Calculator 14
I.4 Numerical Methods 17
References 19
Appendix A. The Models as Stochastic Differential Equations 20
Appendix B. Risk Adjustments in Parameters 21
Appendix C. License Agreement 24

This document is being furnished by OptionCity.net for information purposes only to licensed users of the OptionCity Calculator software product. It subject to change without notice. This document is furnished on as "AS IS" basis, with no warranties, whatsoever, express or implied. OptionCity accepts no responsibility or liability for the consequences of its use, and does not claim that it is free from errors. The software described in this document, including trial or demo versions, is furnished under the license agreement in Appendix C. It is against the law to copy or use the software except as specifically allowed under the license. No part of this document may be reproduced or transmitted in any form or by any means, whether electronically or mechanically, including, but not limited to photocopying, recording, or information recording and retrieval systems, without the express written permission of **OptionCity.net, 983 Bayside Cove W., Newport Beach, Ca, USA 92660, Ph: (949)720-9614**
For the latest available updates to the software and its documentation, please visit http://www.optioncity.net

I TUTORIAL

I.1 Overview

Thanks for trying the **OptionCity Calculator**. The program calculates theoretical option values and related quantities based upon certain mathematical models. While the particular models that the program handles are fairly sophisticated, they are still idealizations of the real-world. These idealizations are part of any theoretical option price calculation. Many assumptions are often unstated, both in this guide and in the original source literature. Consequently, you should use the results carefully.

Advanced option pricing models have two major features not present in the original Black-Scholes' theory. One feature is the addition of stock price jump processes. The other is the change from constant volatility to stochastic volatility. In addition, the models are influenced by the risk attitudes and other subjective preferences of investors. In the Black-Scholes theory, jumps, stochastic volatility, and risk attitudes play no role. Unfortunately, the Black-Scholes theory doesn't generate option smiles or skews either, and is based upon a model with other deficiencies. The OptionCity calculator incorporates these advanced features, generates realistic smiles curves, and at the same time usually performs its calculations very rapidly. (If many of these terms are unfamiliar, please see the Glossary which may be found online at www.optioncity.net).

What's New in Version 1.1? The Calculator now supports American-style options under the CVJ model (see below for more details).

The Models

When you start the calculator, you will see a number of tabs across the top: these tabs refer to the different option models that are handled. In tab order, the models are

Model	Guide Abbreviation
Stochastic Volatility	**SV**
Stochastic Volatility + Jumps	**SVJ**
Constant Volatility	**CV**
Constant Volatility + Jumps	**CVJ**

In order of complexity:

The CV model is the well-known Black-Scholes' (1973) model. The first two rows of every tab are the data entry area for the parameters needed for this model.

The CVJ model is the option model created by R.C. Merton (1976). It is sometimes called *the* jump-diffusion model, although there are other types of jump-diffusion models. The model modifies the Black-Scholes model by adding random stock price jumps. The jumps have a log-normal distribution. You enter three parameters to describe this jump process: (i) a mean jump arrival rate, (ii) a mean jump size, and (ii) a volatility about the mean jump size.

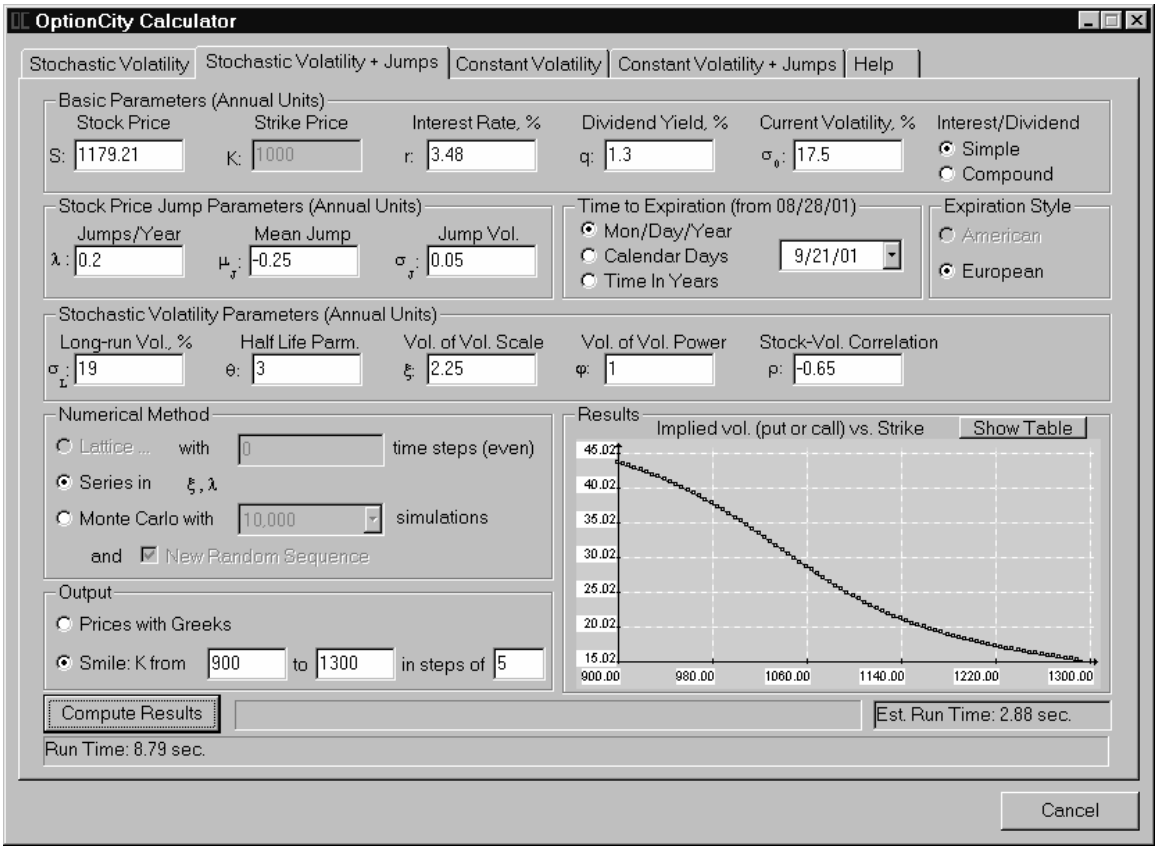
The SV model modifies the Black-Scholes model by making the volatility parameter a random variable. The random or stochastic volatility is mean-reverting, noisy, and correlated with the stock price changes. You enter 5 parameters to describe it: (i) a long-run volatility, (ii) a mean-reversion parameter, (iii) a volatility of volatility scale parameter, (iv) a volatility of volatility power, and (v) a stock-volatility correlation. See below and the book by Lewis (2000) for more technical discussion of this model.

The SVJ model is a combination model that includes both of the above.

All of the models should be interpreted as "risk-adjusted" models (see below and Appendix B).

Hit Compute Results

When you enter the program, sample parameters are already filled in, so you can see how the program works by simply using them. Select what output you want to see: (1) Prices with Greeks or (2) Smile charts/tables. Hit the Compute Results button to get the results (Most computations are short, but the running time is estimated for you and you can always stop a running computation for any reason). For example, switch to the SVJ tab, select the Smile under Output, and hit Compute Results. After a couple of seconds (this is machine dependent) you should see this Smile chart, similar to the following, but with different parameters:



You always have a choice of numerical methods

Notice that you have a choice of numerical methods. In the case above, there are two choices: a Series method, and a Monte Carlo method. A Lattice method, which is available for the SV model, is not available for this model and is grayed-out. Try different numerical methods to get a feeling for their run times on your machine. The "Monte Carlo" method generates output with estimated errors, while the other methods do not have error estimates.

The CV and CVJ models for European-style option have closed-form solutions (i.e., these solutions are known exactly in terms of well-known functions). The more sophisticated models, like the SV and SVJ models (or any case of American-style options), do not have closed-form solutions. Consequently, they must be solved with some numerical method and the numerical method becomes a source of error. In many practical cases, the default numerical results are accurate to a penny or better, but this is not guaranteed. For this reason, the program allows most computations to be cross-validated with at least one other numerical

method. In addition, two of the numerical methods can be run with greater accuracy by adjusting an input value. For example, you can increase the number of time steps in the Lattice methods, and increase the number of simulations in the Monte Carlo method. As a general rule, the "Series" method runs fastest, but needs to be cross-validated, especially if the time to expiration is larger than a few months. The Numerical methods that we use are briefly described in Section 4.

It's possible that a numerical method may fail; this can happen with some extreme parameter values and may happen for some other reason. When a method fails, then the results under Prices with Greeks show a long dash (--), meaning 'Not Available', in one of the output boxes. Similarly, an NA will appear in the Smile chart table of results. If the Smile chart routine is unable to compute enough implied volatilities to draw a graph, then a blank graph will display.

Some strike prices in a Smile chart may have an NA for the Implied Volatility because they are too far from the money and valid Implied Volatilities are impossible to compute. If this happens, try narrowing your range of strikes to get a better looking chart. Or try another numerical method.

Similarly, if some value in the Prices with Greeks output is NA, you might try another numerical method. Be sure to check your input parameters: perhaps one of them is not what you intended.

If you think a numerical method is failing unexpectedly or see some other bug, please email a short description to support@optioncity.net. We appreciate your feedback, comments, or suggestions.

The "Prices with Greeks Output"

The 'Greeks' are the derivatives (or sensitivities) of the option prices with respect to various parameters. We use fairly standard labeling for all of these but one; the particular derivatives are also printed next to the output labels. For example, 'delta' is the derivative of the option price with respect to the stock price. In other words, it's the change in the option (model) price divided by the change in the stock price, as the latter becomes very small. The one 'non-standard' output is labeled the 'hedge ratio"; this is the number of options you need (long or short, we ignore the sign) to hedge the (diffusion part) of the stock price risk if you own one share of stock. For the Black-Scholes model, this is the reciprocal of the "delta". But for the models with stochastic volatility and jumps, there's an extra term involving the volatility. The specific equation for it may be found in Appendix A. When you have stochastic volatility and jumps, the interpretation is that the hedge ratio is how many options you need (to buy or sell) to remove the "Brownian motion" part of the stock price noise (this may be thought of as the "normal market noise"). By "number of options", we are counting one option contract as "100 options". But, even when you have hedged away the normal market noise, there still remains the risk of stock price jumps and changes in volatility.

Exponential notation. Sometimes you may see a result print like this: 7.37e-005
This means 7.37×10^{-5} = .0000737. For example, the Monte Carlo standard errors are often small like this.

The models are "risk-adjusted"

The program computes the theoretical option prices based upon the various models we have discussed. If you are familiar with option pricing models, you probably know that there are two types of random process for any traded security price. The first type is called the "objective" or "statistical" process. The statistical process is the (idealized) random process actually followed by the stock or other underlying security. That is what you are working with when you analyze a time series of stock prices.

The second type is variously called the "risk-neutral", "risk-adjusted", or "martingale-pricing" process. The risk-adjusted process determines option prices. All of the models that are used in this program, and hence their parameters, should be interpreted as risk-adjusted. For most of the parameters, there is no distinction between their risk-adjusted values and their statistical values. However, four parameters can differ: the "Jumps/Year", "Mean Jump", "Long-run Volatility", and "Half-life Parameter". Since you, the user, must supply your own parameters, this distinction may matter to you if you are trying to estimate parameters from a time series of stock prices. To learn more about this, see Appendix B.

II.2 The Option Models and their Parameters

This section explains the parameters of each of the 4 option models, and how they should be entered into the program. For users familiar with Stochastic Differential Equations (SDEs), see Appendix A for a concise (and exact) description of the underlying models. We use a somewhat less technical description in this section. We show an approximate equation describing how the stock price changes (after any risk-adjustment) over a small time step. These equations have the parameters which must be entered into the program.

We use the following notation:

- S_t is the stock price at time t.
- Δt is a small time increment. (pronounced "delta-t")
- ΔS_t is the change in the stock price over Δt. (pronounced "delta S sub t").
- Z_t is a standard normal random variable (it has mean 0, variance 1), which is independently drawn at each time step.

CV: the Constant Volatility model

This is the Black-Scholes model. Over a small time step, the stock price changes by:

$$\frac{\Delta S_t}{S_t} = (r - q)\Delta t + \sigma_0 Z_t \sqrt{\Delta t}\,, \tag{1}$$

where r is an interest rate, q is a dividend yield, and σ_0 is the constant volatility.

To Enter Parameters.
Parameters are entered under the input area labeled Basic Parameters.
General: convert any fractions to decimals. (Example: enter 3 5/8 as 3.625).

S is the current stock price[1].

K is the option strike (exercise) price.
If you have selected Smile output, then the data entry box for K is grayed-out (not used). Instead, enter the range of strikes next to the Smile selection in Output.

r is an interest rate. (Enter an annualized rate in percent).

Example: enter 5.35 for a 5.35% rate. Internally, the program uses $r = 0.0535$. This rate should be a (midpoint) rate for borrowing/lending (in the local currency) that matures close to the option's expiration date. For example, for options quoted in USD, enter a T-bill or Eurodollar rate with the right maturity. There are two input formats: compound (the default) and simple, selected from a radio button on the right. The number that appears in Equation (1) is the (continuously) compounded rate r; this form is used internally. However, quoted rates are simple rates R and you may enter a simple rate. If you select the Simple input format, then internally the program divides your entry by 100, and then converts it to r using $(1+R) = e^r$.

q is a dividend yield (Enter an annualized yield in percent).
Example: enter 1.05 for a 1.05% yield, which the program converts to $q = 0.0105$. This yield should be an approximation to the annualized yield that an owner of the security or index would receive over the time period until the option expires. The same simple/compound conventions apply.

σ_0 is the constant volatility. (Enter an annualized volatility in percent).
Pronounced "sigma sub zero". Example: enter 25 for a 25% annualized volatility, which the program converts to $\sigma_0 = 0.25$.

Time to expiration. (Enter the time to expiration in any of 3 time formats).
A calendar year has 365 days. Entering calendar days = T is identical to entering Time in Years = (T/365). Calendar days may have fractional parts. You can select an expiration date from the calendar control (select Mon/Day/Year and then select the drop-down date box). For the calendar, selecting today's date is treated as 0 days to expiration, tomorrow's date is 1 day to expiration, etc. U.S. equity options usually expire on the Saturday following the third Friday of any month, or one day earlier. (Trading ceases one day before expiration).

---

[1] *Data entry hint*: output option values are shown to 3 decimal places. If you want to see more digits, just enter the stock price and strike price both multiplied by 10 or 100, etc. Then, divide the output option price by the same factor. (Any numerical errors remain unchanged).

Expiration Style.
American-style options may be exercised at any time;
European-style options may be exercised only on expiration.

CVJ: the Constant Volatility + Jumps model
This is the Black-Scholes model with lognormal jumps added.
Over a small time increment, the stock price changes are given by:

$$\frac{\Delta S_t}{S_t} = (r - q - \lambda k)\Delta t + \sigma_0 Z_t \sqrt{\Delta t} + \begin{cases} e^x - 1; & \text{when there is a jump} \\ 0; & \text{otherwise} \end{cases}, \tag{2}$$

The jump amplitude x is a random variable, independently drawn at each jump time from a normal distribution with mean μ_J and standard deviation σ_J. When a jump occurs, the stock price changes $S_{t-} \to e^x S_t$, where t^- means the time just before the jump. The constant $k = \exp(\mu_J + \sigma_J^2/2) - 1$ is the mean jump size (see below).

Parameter entry.
The Basic Parameters are the same as the CV model. In addition, you must enter the three Stock Price Jump Parameters: λ, μ_J, and σ_J . In addition, you may (but are not forced to) enter two numerical method parameters for the case of American-style options. These are: (i) the number of time steps and a parameter (ii) $x_{max} = \log(S_{\max}/K)$, which sets the width of a discrete lattice in term of the maximum stock price and the strike *K*.

λ is the expected jump arrival frequency.
(pronounced 'lambda', annualized units of 1/years).
The jump arrival frequency is the expected number of jumps/year For example, if you expect one jump every month on average, then enter 12. If you expect one jump every 4 years on average, then enter 0.25.

μ_J is a mean logarithmic jump size over a long series of jumps.
(pronounced "mu sub J"). This number is entered as a decimal (example: enter
-0.25 for a –25% logarithmic jump) and can be any positive or negative number or zero.

σ_J is the volatility of *x* about the mean jump. (pronounced "sigma sub J").
This number is entered as a decimal (example; enter 0.20 for a 20% uncertainty about the mean jump) and must be positive or zero.

Converting logarithmic jump sizes to percentage jump sizes. It's easier to think about jumps in percentage terms. The percentage mean jump size is given by the constant *k* above, times 100. For example, if (on average) jumps cause a stock price to fall by 25%, then $k = -0.25$. If you want *k* to have a particular value instead of μ_J, then enter $\mu_J = \ln(1+k) - (\sigma_J^2/2)$, where 'ln' is the natural log. For example, suppose

$\sigma_J = 0.10$ and you want $k = -0.25$. To get it, compute $\mu_J = \ln(0.75) - \frac{1}{2}(0.10)^2 \approx -0.293$. Then, enter $\mu_J = -0.293$. The value for *k* is printed in the status line at the bottom of the tab.

The *implicit total long-run volatility.* It's useful to define a total long-run volatility σ_{total}, which is a quantity affected by risk-attitudes. Bates (1991) calls this the "implicit long-run volatility" and we have added 'total'. It's *implicit* because it's not directly measurable from the stock price series. Nevertheless, it can be inferred from option prices and is given by

$$\sigma_{total} = \sqrt{\sigma_0^2 + \lambda(\mu_J^2 + \sigma_J^2)}$$

This quantity should not be confused with the Black-Scholes' implied volatility. The total volatility is seen to be the sum of two terms: one due to the Brownian motion and one due to the jumps. In a world where people were "risk-neutral", then σ_{total} would equal the total volatility that would be observed for the stock price series over a very long time horizon. However, when people are risk-averse, these two quantities will differ.

SV: the Stochastic Volatility model

In the stochastic volatility model, the volatility $V_t = \sigma_t^2$ is random. The volatility follows a mean reverting process with its own noise term:

$$\begin{aligned} \frac{\Delta S_t}{S_t} &= (r-q)\Delta t + \sigma_t Z_t^{(1)}\sqrt{\Delta t} \\ \Delta V_t &= (\sigma_L^2 - V_t)\theta\Delta t + \xi V_t^{\varphi} Z_t^{(2)}\sqrt{\Delta t} \end{aligned} \quad . \tag{3}$$

In Equation (3), $Z_t^{(1)}$ and $Z_t^{(2)}$ are two standard normal variates, with correlation ρ. These correlated normal variates are independently drawn at each time step.

Parameter entry.

The stock price parameters are entered as before under Basic Parameters. However, the volatility entry σ_0 is interpreted differently than under the CV tab. Instead of being a constant volatility, σ_0 is now treated by the program as the "current volatility". The current volatility is an estimate of what the annualized volatility is at the point in time that you are evaluating the option

To better explain the model, we will discuss it when the time step is one day. Suppose that you have entered today's volatility σ_0 under Basic Parameters. The volatility is random and jumps around just like the stock price. For example, tomorrow's volatility (call it σ_1) will be different. To see how the model describes the changing volatility, put the current volatility into the second equation in (3). Note that $\Delta V_t = V_1 - V_0 = \sigma_1^2 - \sigma_0^2$. On the next day, the model says:

$$\sigma_1^2 = \sigma_0^2 + (\sigma_L^2 - \sigma_0^2)\theta\Delta t + \xi\,\sigma_0^{2\varphi} Z\sqrt{\Delta t} \quad , \tag{4}$$

where we have just written *Z* for the random normal variable. Equation (4) says that there are two effects that will make tomorrows volatility different from today's. The first is the term $(\sigma_L^2 - \sigma_0^2)\theta\Delta t$, this is the

volatility drift. It describes a mean reversion toward a *Long-run* (diffusion) *Volatility* σ_L. Let's ignore the last term for the moment and just concentrate on the meaning of:

$$\sigma_1^2 = \sigma_0^2 + (\sigma_L^2 - \sigma_0^2)\theta\Delta t$$

If the initial volatility σ_0 is less than σ_L, then the whole volatility drift term is positive (when θ is positive). This term then causes tomorrow's volatility to increase, drifting toward the long-run value. If the initial volatility σ_0 is greater than σ_L, then the volatility drift term is negative and tomorrows volatility will decrease, again drifting toward the long-run value σ_L. If the initial volatility σ_0 is exactly equal to σ_L, then there will be no change due to the drift. To summarize:

σ_L is the long-run volatility (annualized units, in percent)[2]

We see that the volatility is drifting toward the long-run value. How rapidly is this occurring? That is described by the *Half-life Parameter* θ, also known as the mean-reversion parameter. Notice that θ appears in (3) multiplying Δt. This product is a pure number with no dimensions. Of course Δt has the dimensions of time (years). So θ has the dimensions of 1/time (per year) and $1/\theta$ has the dimensions of time (years). The meaning of $1/\theta$ is that it measures approximately half the time (in years) it takes for the volatility to drift from its current value to its long-run value. We say $1/\theta$ is the 'half-life' and θ is the 'half-life parameter'.

Here's an example. Suppose the S&P500 index has a long-run volatility of 20% per year but the market becomes very volatile and the current volatility jumps up to 40% per year. Then, 2 months later (2/12 *years*), the volatility has fallen back to the long-run value. If that's typical, then the half-life $1/\theta \approx 1/12$ years. So the half-life parameter $\theta = 12$ (per year). You enter 12. If the half-life is two weeks (2/52 years), then $\theta = 26$. If the half-life is 6 months, then $\theta = 2$ on so on. To summarize:

θ is the Volatility Half-life Parameter (annualized units, per year)[3].

The remaining term in Equation (4), $\xi\,\sigma_0^{2\varphi} Z\sqrt{\Delta t}$, describes the random part of the volatility changes. We will call the whole term the "volatility of volatility" term. We will now discuss the different parts it.

---

[2] Some fairly technical points. (1). When $\varphi \le 1$, the **total** long-run volatility is now the sum of two pieces: $\sigma_L^2 + \lambda(\mu_J^2 + \sigma_J^2)$, similar to the CVJ model, except that we have replaced the constant (diffusion part of the) volatility by the (diffusion part of the) long-run volatility. This value is printed in the status line at the bottom. (2). Unfortunately, things are more complicated when $\varphi > 1$. In that case, the parameter σ_L^2 should not strictly be called the (diffusion part of the) long-run volatility because it's not equal to the mathematical expectation of V_t. Finally, remember that these parameters are affected by preferences.

[3] The program allows you to enter negative values for θ. In that case, there is no mean reversion, but mean *aversion.* This is sometimes useful when fitting market prices or for other reasons.

Since Z is a normal variable, the whole term may be positive or negative. In fact, you can visualize possible values for it by picturing the familiar bell-shape of the normal distribution. Even though the volatility has a drift towards a long-run value (mean-reverting), this drift is interrupted by the unpredictable noise that this term describes.

For example, take the previous example, where the S&P500 volatility has jumped to 40% versus a long-run value of 20%. Although we expect the volatility to eventually decline, tomorrow's volatility might well be higher still, say 45%. That's the unpredictable component, or the volatility of volatility, at work. Now there are two parameters in the volatility of volatility term that we need to explain. First is the *Volatility of Volatility Scale Parameter* ξ (written and hard to pronounce as "ksi"). Second is the *Volatility of Volatility Power* φ (pronounced "phi").

For the moment, suppose $\varphi = 1$. In that case, then ξ is the volatility of volatility (scale) in exactly the same way that σ is the volatility (scale) for the stock price. For example, if $\sigma = 0.50$, then that means very crudely that the stock price can easily vary by 50% during the course of a year[4]. Similarly, if $\xi = 0.50$, then the volatility itself can easily vary by 50% in a year. To take our S&P500 example, if the volatility starts out at the beginning of the year at $\sigma_0 = 20\%$, then $\xi = 0.50$ means that a 50% increase in volatility, to $\sigma = 30\%$ or more is quite possible. (About a 1/6 chance, in fact.) Actually, typical estimates for ξ (again with $\varphi = 1$) for the S&P500 are in the range 0.50 to 2.50, so you can see that volatility itself is very volatile. A value of $\xi = 1$ means that the volatility has a good chance to double over a year and to double even more rapidly for larger values of ξ.

Our model is flexible, and you may enter any value of φ greater than or equal to 1/2. The special case $\varphi = 1/2$ corresponds to an influential stochastic volatility model developed by S. Heston (1993). While probably not the best fit to the marketplace, the special case has an exact solution[5].

If you are valuing options on a certain security, then if you change φ, you should also be changing ξ. For example, for the S&P500, we noted that typical estimates for ξ (with $\varphi = 1$) are in the range 0.50 to 2.50. If, instead, you use $\varphi = 1/2$, then your estimates for ξ should be lower: for example in the range, 0.1 to 0.5. In the other direction, if you use a value for φ greater than 1, then your estimates for ξ should be higher.

The case $\varphi = 1$ is very useful because the model corresponds closely to a well-established theory of volatility called GARCH Modelling, developed by R. Engle, T. Bollerslev, and others. That correspondence means that you can use GARCH software to estimate the SV parameters[6].

---

[4] Or more accurately, the log of the stock price will vary by 50% or less in 2 years out of 3.

[5] Our numerical routines (see Sec. 3) treat this case no differently than any other value for φ.

[6] A practical solution to parameter estimation for the OptionCity Calculator is. (1) Use available GARCH software to estimate the stochastic volatility parameters; (2) Fit the jump parameters from the option market. OptionCity.net may provide software tools to assist with this. For more about GARCH models, see Bollerslev, Chou and Kroner (1992).

What's the best value for φ? When φ is allowed to float and parameters for this SV model are estimated or fitted to the options market, then values somewhat higher than 1 are suggested. However, the inclusion of jumps can change this.

Finally, the last SV parameter is the correlation ρ (pronounced "rho"). This parameter measures the correlation between changes in the stock price and changes in the volatility. It has to be a number between –1 to 1. [Actually, there is a further limitation, when you are using the Lattice numerical method, to the range $-\sqrt{3/4}$ to $\sqrt{3/4}$, where $\sqrt{3/4} \approx .866$; see Leisen(2000)].

If you have traded options, then you have probably seen the common phenomenon that when the market falls, index volatility usually (but not always) goes up and vice-versa. This behavior is modeled by a negative value for ρ and is sometimes called the "leverage effect". Negative values for ρ help impart skewness to the smile charts. These charts have a negative slope (close-to-the-money) in the SV model when ρ is negative. They have a positive slope (close-to-the-money) when ρ is positive. And, when ρ is zero, the smiles are more symmetrical, similar to parabolas for strikes close to the money. To summarize:

ξ is Volatility of Volatility Scale Parameter.
Enter any positive number or zero.

φ is the Volatility of Volatility Power.
Enter any number greater than or equal to 1/2.

ρ is the Stock-Volatility Correlation.
Enter any number between –1 and 1.
(or any number between $-\sqrt{3/4}$ to $\sqrt{3/4}$ for the Lattice method).

Other SV parameters: The two numerical method parameters σ_{MIN} and σ_{MAX} are generally best left alone, but are described further under the Section II.4, "Numerical Methods".

IV. SVJ: the Stochastic Volatility + Jumps model

This is the most flexible and probably the most useful model. It's just like Equation (3), except that the stock price can jump as in Equation (2). There are no new parameters introduced. Each parameter has the same meaning and data entry restriction as described above.

Note that although the parameters have the same meaning, if you are fitting them to the options market, the "best fits" will have different values under the different models. That's because both jumps and stochastic

volatility can contribute to the option skews in the SVJ model, but only one of them contributes in other models.

II.3 Working with the Calculator

Here's a simple example showing how to work with the Calculator to generate fairly realistic smile charts. We used the Calculator to value put options on the S&P500 index, which closed at 1179.21 on August 27, 2001. These are European-style options. The tables below show closing bid and ask on the options. (The calculator analysis was actually run on August 28, so the calendar control has measured the time from that date). We used a dividend yield of 1.3% across all expirations. For interest rates, we used a different value for each expiration: namely, the Aug. 27, 2001 closing values for the Eurodollar futures, taken from the Wall Street Journal. There is a future that corresponds to each of the option expiration months in the tables. These yields and rates are simple rates, not continuously compounded, so we selected the "Simple" option.

How we worked. First, we used the CV tab to get the Black-Scholes' implied volatilities (at the bid-ask average). Then, we moved to the SVJ tab. For simplicity, we fixed the Volatility of Volatility power at 1, since this corresponds to a case where we had some ideas about parameters. We started adjusting the parameters from the default values, not in any systematic way, but just trying to improve the fits to the September 2001 and June 2002 expirations. When comparing results for the Sept 2001 expiration, we found that the Series method results were virtually identical to the Monte Carlo method results. Since the Series is faster, we used that. For that expiration, most of our parameter fitting adjustments were made to the current volatility and the jump parameters.

For the June 2002 expiration, we saw that the differences between the Series method and the Monte Carlo method were larger, certainly greater than 1 penny. In cases like that, its probably best to stick with the Monte Carlo method, which we ran with 100,000 simulations. For that expiration, most of our parameter adjustments were made to the first 3 volatility parameters. After moving back and forth a couple of times between those two expirations, we settled on the parameters shown in the screenshot on page 7. Then, we filled in the other tables using that set of parameters. Note that the numbers in parenthesis are the Monte Carlo estimated errors. When no error is shown, it's less than the last digit shown.

SPX Index Options: Market Quotes and Model Prices, August 27, 2001

Option Expiration: Sept. 21, 2001 Eurodollar rate: 3.48%

			Market			OptionCity SVJ Model	
Type	Strike	Vol-ume	Bid	Ask	Implied Volatility	Price	Implied Volatility
put	900	0	0.15	0.20	42.2	0.19	42.6
put	1050	2183	1.65	1.90	28.8	1.88	29.2
put	1150	4708	11.50	11.80	20.6	11.26 (.02)	20.2
put	1175	1885	18.70	20.20	18.9	19.47 (.03)	18.9
put	1200	212	31.50	32.50	17.7	32.19 (.03)	17.9

Option Expiration: Dec. 21, 2001 Eurodollar rate: 3.54%

			Market			OptionCity SVJ Model	
Type	Strike	Vol-ume	Bid	Ask	Implied Volatility	Price	Implied Volatility
put	900	51	2.90	3.60	29.4	2.79 (.02)	28.6
put	1050	351	14.10	15.60	24.2	14.69 (.05)	24.1
put	1150	2971	35.90	37.90	20.6	37.19 (.07)	20.7
put	1175	174	45.40	47.40	20.0	46.32 (.07)	20.0
put	1200	102	56.60	58.60	19.3	57.32 (.08)	19.2

Option Expiration: Mar. 15, 2002 Eurodollar rate: 3.67%

			Market			OptionCity SVJ Model	
Type	Strike	Vol-ume	Bid	Ask	Implied Volatility	Price	Implied Volatility
put	900	0	6.70	7.70	27.1	6.50 (.04)	26.5
put	1050	0	23.60	25.60	23.2	24.34 (.07)	23.2
put	1150	100	49.40	51.40	20.7	50.92 (.09)	20.9
put	1175	0	58.70	60.70	20.1	60.35 (.10)	20.3
put	1200	40	69.30	71.30	19.5	71.12 (.10)	19.8

Option Expiration: June 21, 2002 Eurodollar rate: 3.99%							
			Market			OptionCity SVJ Model	
Type	Strike	Vol-ume	Bid	Ask	Implied Volatility	Price	Implied Volatility
put	900	0	11.80	13.30	26.3	10.94 (.06)	25.3
put	1050	0	33.00	35.00	22.9	33.28 (.09)	22.7
put	1150	101	60.90	62.90	21.0	62.21 (.11)	21.0
put	1200	20	80.90	82.90	20.1	82.38 (.12)	20.2

Option Expiration: Dec. 20, 2002 Eurodollar rate: 4.77%							
			Market			OptionCity SVJ Model	
Type	Strike	Vol-ume	Bid	Ask	Implied Volatility	Price	Implied Volatility
put	900	0	20.10	23.10	25.9	17.71 (.08)	24.3
put	1050	0	46.50	49.50	23.3	44.30 (.11)	22.4
put	1150	0	75.70	78.70	21.9	74.24 (.13)	21.3
put	1200	0	95.00	98.00	21.3	93.64 (.14)	20.8

Most of the fits are pretty good and were obtained without too much effort. However, the model prices are below the market for most cases of the 900 strike and the Dec, 2002 expiration. Only one of those contracts traded, so perhaps those deviations are not too important. In any event, the fits could be improved by a more systematic procedure, like minimizing the squared errors across all of the options.

Interpretation of results. Here is an interpretation of these implied parameters. The market (on this date) is expecting a mean jump of about –22% to occur about once every 5 years in the SPX, all on a risk-adjusted basis. (This figure comes from calculating the mean jump size *k,* as explained in Section 2, under the CVJ model). In addition there is an implied –0.65 correlation between the (non-jump related) S&P 500 Index returns and the volatility changes. Finally, from footnote 2, the implicit long-run volatility is the sum of two pieces: $\sigma_L^2 + \lambda(\mu_J^2 + \sigma_J^2)$. If you take the square-root of this expression, using the parameters in the screenshot, you get about 22% per year. That value is plausible and coincidentally the same as the mean jump size: there's no reason for that, it's just a coincidence. Remember that the implicit long-run volatility is affected by preferences and risk attitudes, so the fact that it's higher than the historical long-run volatility of the SPX is not a cause for concern.

II.4 Numerical Methods

CV: The closed-form European-style results are the Black-Scholes model with a continuous dividend yield *q*. The calculations use a numerical approximation to the cumulative normal function; this approximation is accurate to an absolute error of 7.5 x 10^{-8} or less. The CV lattice method is the Cox-Ross-Rubinstein binomial method with a Control Variate correction, as described in Hull and White (2000) for example. This lattice method is invoked whenever an implied volatility is needed for an American-style option on any tab.

CVJ: For European-style options, the closed-form solution is Merton's (1976) power series solution. For American-style options a lattice method has been created, which is explained in more details in Lewis (2003). This discretization uses a time step $\Delta t = T / N$, where T is the time to expiration and N is the number of time steps. By choosing "auto", an internal rule is used to determine *N;* how you may override it. Similarly, the lattice makes use of a width parameter $x_{\max} = \log(S_{\max} / K) = \log(K / S_{\min})$, where $S_{\max}$ and $S_{\min}$ are maximum and minimum stock prices and K is the strike price. You may also accept the internal rule or override this selection.

SV: The SV Lattice method is an OptionCity.net proprietary extension of the lattice method described in Leisen (2000). The extension handles the more general SV model and runs a lot faster. The lattice is restricted to value of volatility (σ) lying between the two parameters σ_{MIN} and σ_{MAX}. Generally, these values should be left alone as they usually represent good bounds. However, advanced users may want to adjust these values if they know, or suspect, that the volatility process (over the time period to expiration) will have some significant probability of reaching volatility values that lie outside the bounds shown. (Be warned that increasing the bounds may decrease the numerical accuracy at a given lattice size).

The Series in ξ method first calculates a power series for the Implied Volatility through $O(\xi^2)$, as described in Lewis (2000, Chs.3,5). Call the power series values: 'Vimplied'. This power series was developed for Euro-style options. To get Euro-style prices, you feed Vimplied into the Euro-style Black-Scholes formula. However, we also use Vimplied values in an approximation for American-style option prices. The approximation is to feed the same (Euro-style) Vimplied into the Amer-style CV lattice method, described above. Under this approximation, the SV smile charts will be exactly the same for either Euro-style or Amer-style options. (You can check the accuracy of the approximation by comparing against the SV Lattice method). The option prices themselves will differ in the two cases, and will each satisfy the appropriate boundary conditions for Amer-style vs. Euro-style prices. (For example, Euro-style put prices can lie below the parity value of $K - S$, but not Amer-style put prices).

The SV Monte Carlo method is the Monte Carlo Mixing technique described in Lewis (2000, Ch. 4). Note that you may choose the number of Monte Carlo simulations. The time step Δt has a default value given by solving: $\max(\sigma_0^2, \sigma_L^2)\Delta t = 10^{-4}$. This usually works well, but if you would like to change it, just enter a new "Scale" factor in the text entry box labeled: **Scale dt $\rightarrow$ dt/[ ]**. For example, if you enter a 10, then Δt is reduced by a factor of 10. Of course, the Monte Carlo evaluations will then take 10 times longer to run.

SVJ: Only Euro-style options are handled. For those, the Series and Monte Carlo methods are similar to the SV methods, but extended to incorporate the jump process.

References

Bates, David S. (1991): *The Crash of '87: Was it Expected? The Evidence from Options Markets*, Journal of Finance, Vol. 46, July, pages 1009-1044.

Bollerslev, T., R.Y. Chou and K.F. Kroner (1992): *ARCH Modeling in Finance. A Review of the Theory and Empirical Evidence*, Journal of Econometrics, Vol. 52, pages 5-59.

Friend, I. and M. Blume (1975): *The Demand for Risky Assets,* American Economic Review, Vol. 65, pages 900-922.

Hull, John C (2000): *Options, Futures, and Other Derivatives*. Fourth edition. Prentice Hall, New Jersey.

Leisen, Dietmar P.J. (2000): *Stock Evolution under Stochastic Volatility: A Discrete Approach.* Journal of Derivatives, Vol. 8, No.2, pages 9-27.

Lewis, Alan L. (2000): *Option Valuation under Stochastic Volatility*, Finance Press, Newport Beach, California. See also *Option Valuation under Stochastic Volatility II: Jumps and Exotics* (forthcoming 2006).

Lewis, Alan (2002): *Fear of Jumps*, Wilmott magazine, Dec. 2002, 60-67.

Lewis, Alan (2003): *American Options under Jump-diffusions: an Introduction,* Wilmott magazine, March 2003.

Merton, R.C.(1976): *Option Pricing When Underlying Stock Returns are Discontinuous*, Journal of Financial Economics, Vol. 3, Jan-Mar, pages 125-144. Reprinted as Ch. 9 in *Continuous-Time Finance*. Basil Blackwell, Cambridge Mass (1990).

Appendix A:
The Models as Stochastic Differential Equations (SDEs)

For users familiar with SDEs, here are the models written in that form. We use dB_t and dW_t as symbols for two correlated Brownian motions, with correlation ρ. Also, dQ_t is a symbol for an independent compound Poisson process, with intensity λ and jump amplitude $e^x - 1$, where $x \sim N(\mu_J, \sigma_J^2)$. ($x$ is normally distributed with the mean and variance shown). We also use $V_t = \sigma_t^2$ and the constant $k = \exp(\mu_J + \sigma_J^2/2) - 1$. In general, any quantities without a t subscript are constant parameters. With that notation, the risk-adjusted models are:

CV:
$$\frac{dS_t}{S_t} = (r-q)dt + \sigma_0\, dB_t$$

CVJ:
$$\frac{dS_t}{S_t} = (r-q-\lambda k)dt + \sigma_0\, dB_t + dQ_t$$

SV:
$$\begin{cases} \dfrac{dS_t}{S_t} = (r-q)dt + \sigma_t\, dB_t \\ dV_t = (\sigma_L^2 - V_t)\theta\, dt + \xi V_t^{\varphi} dW_t \end{cases}$$

SVJ:
$$\begin{cases} \dfrac{dS_t}{S_t} = (r-q-\lambda k)dt + \sigma_t\, dB_t + dQ_t \\ dV_t = (\sigma_L^2 - V_t)\theta\, dt + \xi V_t^{\varphi} dW_t \end{cases}$$

Hedging. For the CV model, if you own $n^{(1)}$ shares of stock, then you can completely eliminate all risk with an offsetting position of $n^{(2)}$ options, where

$$n^{(2)} = \frac{n^{(1)}}{F_S}.$$

This is well-known 'delta-hedging', where F_S is the partial derivative of the option price with respect to the stock price (the delta). When volatility is stochastic, you can't hedge all the risks, but you can still eliminate the Brownian noise dB_t as a position risk. Again you take a position in $n^{(2)}$ options, where now

$$n^{(2)} = \frac{n^{(1)}}{\left(F_S + \dfrac{\rho\,\xi\, V^{\varphi-1/2}}{S} F_V\right)} = \frac{n^{(1)}}{\left(\text{Delta} + \dfrac{100\rho\,\xi\,\sigma^{2\varphi-2}}{2S}\text{Vega}\right)}$$

See Lewis (2000, page 18). The absolute value of $n^{(2)}/n^{(1)}$ is defined to be the "hedge ratio" and is shown in the output of the "Prices with Greeks". Section 2 contains more information about parameter interpretations and data entry restrictions.

Appendix B: Risk Adjustments in Parameters

There are four input parameters related to drifts. These may be influenced by the risk attitudes of investors. Pair A is the Stochastic Volatility Parameters: σ_L (the long-run volatility) and θ (the volatility half-life parameter). Pair B is the Stock Price Jump Parameters: λ (mean jumps/year) and μ_J (mean logarithmic jump size).

For an example of how this works, consider λ in the models with stock price jumps. Suppose you are trying to value the S&P 500 index options and you are using the CVJ model or the SVJ model. For simplicity, suppose the jumps as pretty rare events that correspond to a market crash (like the Oct. '87 crash) or a smaller mini-crash. These jumps are rare, but large and negative when they happen. Statististically, let's say market crashes occur only once every 10 years as an example: this means the statistical value for λ is only 0.10. But, because people are very risk-averse, they may rationally price options "as if" crashes occur once every 4 years; this means λ is 0.25. It is this latter 'risk-adjusted' value that you should use in the program. Below, we give a simple formula that you may find helpful in connecting these two values for λ.

How should you deal with this complication of risk adjustments? It's a fact of life in advanced option models. If this complication didn't exist, options probably wouldn't exist either because they would be "redundant" securities. Here are three approaches that you may want to consider, each with its own strengths and weaknesses. For more discussion about risk adjustments in models with jumps, see Lewis (2002).

One method is to ignore the problem. This can makes sense in cases where the results are insensitive to certain parameters. A rough rule of thumb: the SV drift parameters usually have little effect on option prices at very short times to expiration. So, if that's what you are looking at, don't worry too much about those parameters – worry about the jump parameters. The reverse may be true at very distant expirations. You can test these possibilities by running the program on a case of interest, changing parameters, and seeing if the outputs change enough to matter to you. A disadvantage is that this may work for some, but not all of the 4 drift-related parameters.

A second method is to fit the parameters to the options markets. The advantage of this approach is that you don't have to make difficult estimates for what may be rare events, like jumps, or means, like the volatility drift parameters. Some disadvantages: if you can't estimate these parameters, maybe the market can't either. Perhaps the market estimates you obtain may be unrealistic. Of course, maybe the market is right and the model is wrong!

Finally, a third method is to start with some estimate of the statistical values, but then apply the particular risk adjustments that come from certain models (called 'equilibrium' models). If you use the second method (fitting to the market), then you might want to make sure your results are reasonable by comparing them to those from the third method. The advantage of this method is that it's more of a 'first principles' approach. The disadvantage is that it introduces another source of uncertainty: picking the right equilibrium model. Here's a simple example of this third approach applied to the jump parameters. [See Bates (1991)].

In the example equilibrium model, investors in the aggregate, are averse to risk (they will not pay the mathematical expectation for a risky proposition, but will only pay a discounted price). Investors are described by a single risk-aversion parameter γ ('gamma') which is some number less than or equal to 1. If investors are 'risk-neutral', then they have $\gamma = 1$. According to a classic study in economics, (Friend and Blume, 1975) the value of γ for U.S. households over a long period is on average below zero and probably below –1. Some economists believe in much more negative values. In this model, investors won't invest in stocks unless the overall stock market provides a long-run expected return of $r + (1-\gamma)\sigma^2$, where r is the short-term interest rate, and σ is the volatility (assumed constant in this model). For example, if investors expect the stock market to provide 10% above T-bills on average, with 20% per year volatility, then $0.10 = (1-\gamma)(0.20)^2$, and so $\gamma = -1.5$. Put in your own values for the market return and volatility to see your own (implied) value for γ.

Now, with this model, here is the risk-adjustment formulas for the jump parameters. Suppose the statistical parameters (the ones you would estimate from the time series of stock prices) are $(\hat{\lambda}, \hat{\mu}_J, \hat{\sigma}_J)$ (pronounced lambda hat and so on). First, compute

$$c = \exp\left[-(1-\gamma)\hat{\mu}_J + \frac{1}{2}(1-\gamma)^2\hat{\sigma}_J^2\right].$$

With that, the risk-adjusted parameters $(\lambda, \mu_J, \sigma_J)$, which are the ones you need to input into the program are given by the simple formulas:

$$\lambda = c\hat{\lambda} \qquad \mu_J = \hat{\mu}_J - (1-\gamma)\sigma_J^2 \qquad \sigma_J = \hat{\sigma}_J$$

The jump volatility σ_J is the same with or without a risk adjustment. Here's a numerical example for the others. Take the case again of jumps as rare market crashes. Suppose your estimate is that, statistically, such market jumps occur once every 10 years on average $(\hat{\lambda} = 0.10)$, with a mean logarithmic jump of –25% $(\hat{\mu}_J = -0.25)$ and volatility of 15% about that $(\sigma_J = 0.15)$. Then, with the same value $\gamma = -1.5$ from the above example, we have

$$c = \exp\left[(2.5)(0.25) + (0.5)(2.5)^2(0.15)^2\right] = \exp[0.6953] \approx 2\,.$$

$$\mu_J = -0.25 - (2.5)(0.15)^2 \approx -0.31$$

So, $\lambda = c\hat{\lambda} \approx 2(0.10) = 0.20$. Then you would enter into either the SVJ tab or the CVJ tab the values $\lambda = 0.20$, $\mu_J = -0.31$, and $\sigma_J = 0.15$. With this example, the risk-adjusted jump frequency is once every 5 years on average, with a mean logarithmic jump of –31% and volatility of 15% about that. Risk-averse investors fear losses more than they value gains. Because of that, they rationally price Index options 'as if' the market was going to fall twice as often as it actually does and with a greater average fall when it does. Of course, this is just a particular example.

Appendix C: License Agreement

OptionCity.net's
OPTIONCITY CALCULATOR: End User License Agreement

IMPORTANT: This is an agreement under which you are permitted to install and use the software program "OptionCity Calculator version 1.1" (the Software). You should carefully read the following terms and conditions before using this software. By using the Software, you are consenting to be bound by this agreement. If you do not agree to all of the terms of this agreement, do not use the Software and remove it from your computer.

DISCLAIMER OF WARRANTY
This software and accompanying files and documentation are provided free of charge, on an "AS IS" basis, without warranty of any kind, including without limitation the warranties of merchantability, fitness for a particular purpose and non-infringement. The entire risk as to the quality and performance of the Software is borne by you. In no event shall OptionCity.net be liable to you for any consequential, incidental, or special damages, whether foreseeable or not, including any lost profits or lost savings, even if an OptionCity.net Software representative has been advised of the possibility of such damages, or for any claim by any third party. Consequential damage would include financial losses resulting from errors produced by the program. Should the Software prove defective, you and not OptionCity.net assume the entire cost of any service and repair. You must determine whether the Software sufficiently meets your requirements.

Although reasonable care has been taken in the construction of this Software, OptionCity.net makes no claim that the results are correct, valid or meaningful. You should crosscheck and confirm the results displayed by this program with other sources. At no time should the Software be used or relied upon as the sole analytical tool in any investment decision. Option trading involves many risks; these risks are not removed by the use of any software. You should read "Characteristics and Risks of Standardized Options", available from the Chicago Board Option Exchange and elsewhere.

OptionCity.net is not responsible for problems which may occur as a result of any incompatibility between the Software and any other software or hardware. You assume responsibility for the selection of the Software to achieve you intended purposes, for making backups of your data regularly, and for choosing, maintaining and matching your hardware, operating system software and other applications software.

No oral or written information or advice given by OptionCity.net or an OptionCity.net representative shall create a warranty or in any way increase the scope of this warranty.

GOVERNING LAW

This agreement shall be governed by the laws of the state of California, USA

If any part of this agreement is found void and unenforceable, it will not affect the validity of the balance of the agreement, which shall remain valid and enforceable according to its terms.

This agreement may only be modified in writing signed by an authorized officer of OptionCity.net.

LICENSE GRANT
You may install the software on up to two computers, but you may use the Software on only one computer at a time, and may not use the Software from a network server.

You may make copies of the Software for backup or archival purposes, providing that any copy must contain all of the original Software's proprietary notices. You may not distribute copies of the Software or its documentation to others.

You may not modify, translate, reverse engineer, de-compile, disassemble or create derivative works based on the Software; rent, lease, grant a security interest in, or otherwise transfer rights to the Software; or remove any proprietary notices on the Software.

TITLE
Title, ownership rights, and intellectual property rights in the Software shall remain in OptionCity.net. The Software is protected by the copyright laws and treaties. Title and related rights in the content accessed through the Software is the property of the applicable content owner and may be protected by applicable law. This license gives you no rights to such content.

TERMINATION
This license will terminate automatically if you fail to comply with the limitations described herein. On termination, you must destroy all copies of the Software and Documentation.

LIMITATION OF LIABILITY
Under no circumstances and under no legal theory, tort, contract or otherwise, shall OptionCity.net be liable to you or any other person for any indirect, special, incidental or consequential damages of any character including, without limitation, damages for loss of goodwill, work stoppage, computer failure or malfunction, or any and all other commercial damages or losses.

COPYRIGHT
The Software is the exclusive property of OptionCity.net, and its structure, organization and code are valuable trade secrets of OptionCity.net. This agreement does not grant you any intellectual property rights in the Software.

IF YOU DO NOT ACCEPT ANY TERMS CONTAINED IN THIS AGREEMENT, REMOVE THE PROGRAM IMMEDIATELY.